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Discrete Second Order Sliding Mode Control for Input-Output Model

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***Abstract*—A new discrete second order sliding mode control for the single input single output systems is proposed in this paper. The control law is based on an input-output model. The new control strategy is designed in order to reduce the chattering phenomenon that appears in classical sliding mode control. A numerical simulation example is presented to illustrate the effectiveness of the proposed control. The obtained results shows good performances in term of elimination of chattering phenomenon.**

I. INTRODUCTION

The sliding mode control (SMC) has been widely used in literature. This success is due to its simplicity and robustness against external disturbances and uncertainties [11], [16], [17]. The sliding mode control consists in two steps. The first step is to design a sliding surface along which the process can slide to find its desired final value. The second step is to develop a control law in such away that any state outside the sliding surface is forced to reach the desired sliding manifold in finite time and stay on it.

The evolution of the representative point motion to the origin, assumed to be the equilibrium point, is performed in two phases. The first is called the reachability phase during which the representative point starting from any initial point, reaches the sliding surface in a finite time. The second phase is the sliding mode. In this phase, the representative point slides on the sliding surface until reaching the origin (equilibrium point). Discrete sliding mode controller have been developed mainly using state-space models [1], [5], [7], [8], [12]–[14]. Recently, the use of input-output models in the design of discrete sliding mode control has received some attention [2], [3], [6], [9], [10], [18].

The first work based on input-output model are developed by Pieper and Goheen [15]. Furuta [6] proposed an adaptive control law for the systems with unknown parameters using an input-output model. The work of Pieper, Goheen and Furuta does not include disturbances. Chan [2] suggested a simple discrete sliding mode tracking controller for the systems with model uncertainty and disturbance. Later, Chan [3] formulated a discrete adaptive sliding mode tracking control of a dynamical system with input-output representation in the presence of a bounded disturbance. A robust adaptive quasi-sliding mode controller for a much more general class of discrete input-output systems with unknown parameters, unmodeled dynamics and bounded disturbances is proposed in [18]. In [4], Chen developed a robust adaptive sliding mode controller for the multi-input multi-output systems with unknown parameters and disturbances. A control laws based on the combination of sliding mode control and repetitive control are developed in [5] to multivariable systems modeled with input-output model. In spite of the robustness of the sliding mode control against external disturbances and uncertainties, it has a drawback such as the chattering phenomenon caused by the discontinuous part of the control law [11], [14], [17]. To reduce this phenomenon, many solutions are proposed such as second and high order sliding mode control [12], [13], [17]. The principle of the high order sliding mode control is to force the states to reach the sliding surface and maintain the sliding function s and its $(r-1)$ derivatives to zero such as:

$$s = \dot{s} = \ddot{s} = \cdots = s^{(r-1)} = 0$$

This work proposes a new second order sliding mode control for single-input single-output systems which is based on an input-output model. This control law is able to reject the disturbances and to eliminate the chattering phenomenon.

This paper is organized as follows. In section II, we develope the classical quasi-sliding mode control (SMC). Section III gives a simulation example using this classical SMC. Then, we synthesis a discrete second order sliding mode control for single input single output systems IV. Simulation results are given in section V. Finally, a conclusion is presented.

II. DISCRETE SLIDING MODE CONTROL

Consider the single-input single-output system described by the following model:

$$A(q^{-1})y(k) = q^{-1}B(q^{-1})u(k) + d(k) \tag{1}$$

where $y(k)$, $u(k)$ and $d(k)$ are respectively the output, the input and the disturbance. $A(q^{-1})$ and $B(q^{-1})$ are two polynomials in the unit-delay operator q^{-1} defined as :

$$\begin{cases} A\left(q^{-1}\right) = 1 + a_1 q^{-1} + \cdots + a_{n_A} q^{-n_A} \\ B\left(q^{-1}\right) = b_0 + b_1 q^{-1} + \cdots + b_{n_B} q^{-n_B} \end{cases}$$

It assumed that b_0 is a non-zero constant and the disturbance $d(k)$ is assumed to be bounded such that:

$$\mid d(k) - d(k-1) \mid \leq \varepsilon < \infty \qquad \forall \;\; k \tag{2}$$

The sliding surface is given by the following expression [2], [3], [6], [18]:

$$S(k) = C(q^{-1})(y(k) - r(k)) = C(q^{-1})e(k) = 0 \quad (3)$$

where $r(k)$ is the reference input and $C(q^{-1})$ is a stable polynomial defined as:

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{n_C} q^{-n_C}$$

Consider $F(q^{-1})$ and $G(q^{-1})$ the two polynomials solution of the diophantine polynomial equation:

$$C\left(q^{-1}\right) = A\left(q^{-1}\right) E\left(q^{-1}\right) F\left(q^{-1}\right) + q^{-1} G\left(q^{-1}\right) \quad (4)$$

where

$$\begin{cases} F\left(q^{-1}\right) = 1 \\ G\left(q^{-1}\right) = g_0 + g_1 q^{-1} + \cdots + g_{n_G} q^{-n_G} \\ n_G = sup(n_c - 1, n_A) \\ E\left(q^{-1}\right) = 1 - q^{-1} \end{cases}$$

The sliding mode control law is given by:

$$u(k) = u_{eq}(k) + \ u_{dis}(k) \quad (5)$$

where $u_{eq}(k)$ is the equivalent control law that is obtained for:

$$S\left(k+1\right) = S\left(k\right) = 0 \quad (6)$$

The sliding function at the instant $k+1$ can be calculated as:

$$\begin{aligned} S\left(k+1\right) &= C\left(q^{-1}\right)\left(y\left(k+1\right) - r\left(k+1\right)\right) \\ &= C\left(q^{-1}\right)\left[A\left(q^{-1}\right)\right]^{-1} \begin{bmatrix} q^{-1} B\left(q^{-1}\right) u\left(k+1\right) \\ + d\left(k+1\right) \end{bmatrix} \\ &\quad - C\left(q^{-1}\right) r\left(k+1\right) \end{aligned}$$

By using the equation (4) and without the external disturbances, the sliding function $S(k+1)$ becomes:

$$\begin{aligned} &S\left(k+1\right) = \\ &\left[A\left(q^{-1}\right) E\left(q^{-1}\right) + q^{-1} G\left(q^{-1}\right)\right] \left[A\left(q^{-1}\right)\right]^{-1} B\left(q^{-1}\right) u\left(k\right) \\ &- C\left(q^{-1}\right) r\left(k+1\right) \\ &\quad = E\left(q^{-1}\right) B\left(q^{-1}\right) u\left(k\right) + \left[A\left(q^{-1}\right)\right]^{-1} \\ &\quad G\left(q^{-1}\right) B\left(q^{-1}\right) u\left(k-1\right) - C\left(q^{-1}\right) r\left(k+1\right) \end{aligned}$$

By using the equation (1), the sliding function $S(k+1)$ can be written as:

$$\begin{aligned} S\left(k+1\right) = B\left(q^{-1}\right) E\left(q^{-1}\right) u\left(k\right) + G\left(q^{-1}\right) y\left(k\right) \\ - C\left(q^{-1}\right) r\left(k+1\right) \end{aligned}$$

Then, the equivalent control law is given by:

$$u_{eq}\left(k\right) = \left[B\left(q^{-1}\right) E\left(q^{-1}\right)\right]^{-1} \begin{bmatrix} -G\left(q^{-1}\right) y\left(k\right) \\ + C\left(q^{-1}\right) r\left(k+1\right) \end{bmatrix} \quad (7)$$

The robustness is ensured by addition of a discontinuous term $u_{dis}(k)$ such as:

$$u_{dis}\left(k\right) = -M\, sign\left(S\left(k\right)\right) \quad (8)$$

where $sign$ is the signum function defined as:

$$sign\left(S\left(k\right)\right) = \begin{cases} -1 & if & S\left(k\right) < 0 \\ 1 & if & S\left(k\right) > 0 \end{cases}$$

III. SIMULATION EXAMPLE

Consider the single-input single-output system described as follows:

$$A(q^{-1})y(k) = q^{-1}B(q^{-1})u(k) + d(k)$$

with

$$\begin{cases} A\left(q^{-1}\right) = 1 - q^{-1} + 0.24 q^{-2} \\ B\left(q^{-1}\right) = 1 - 0.5\, q^{-1} \end{cases}$$

The polynomial $C(q^{-1})$ is chosen as:

$$C\left(q^{-1}\right) = 1 - 0.8\, q^{-1}$$

The synthesis parameters are chosen as:

$$M = 0.1, \qquad T_e = 0.01$$

The reference input is chosen as:

$$r\left(k\right) = 2$$

A. Case 1: without external disturbances $d(k) = 0$

The simulation results of classical sliding mode control are shown in figures 1, 2 and 3. Figure 1 shows the evolution of the output $y(k)$ and the desired reference trajectory $r(k)$, figure 2 shows the evolution of the controller $u(k)$ and figure 3 shows the evolution of the sliding surface $S(k)$.

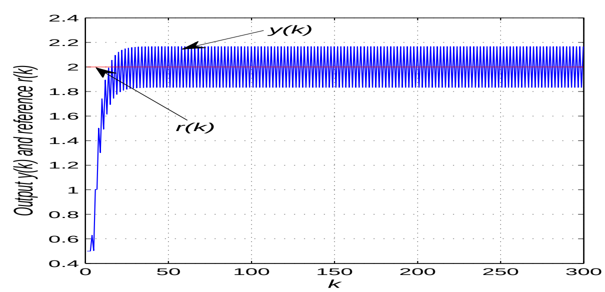

Fig. 1. Evolution of the output $y(k)$ and the desired reference trajectory $r(k)$ DSMC without external disturbances

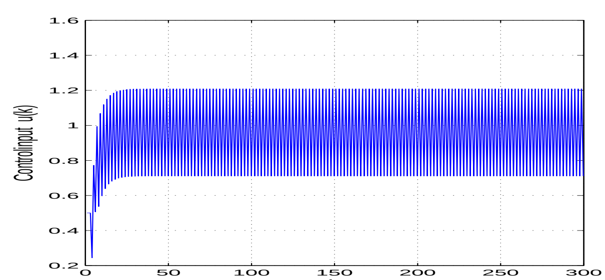

Fig. 2. Evolution of control input $u(k)$ DSMC without external disturbances

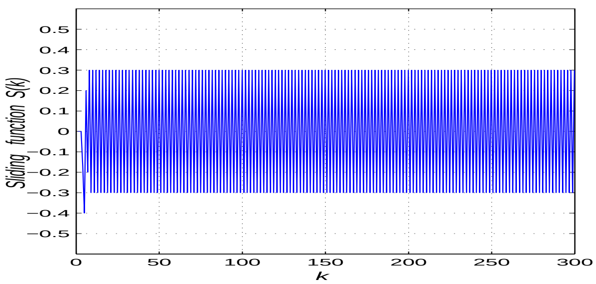

Fig. 3. Evolution of the sliding function $S(k)$ DSMC without external disturbances

It can be seen that the classical sliding mode control (SMC) can not remove the chattering phenomenon.

B. *Case 2: with external disturbances*

In this case the disturbance is chosen as:

$$d\left(k\right) = 0.5 \qquad\qquad if \quad k > 100$$

The evolution of disturbances $d(k)$ is given in figure 4.

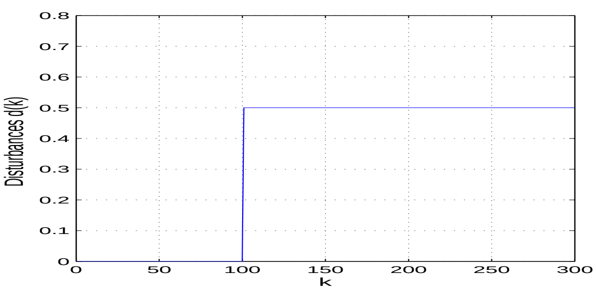

Fig. 4. Evolution of the disturbances $d(k)$

The simulation results of classical sliding mode control are shown in figures 5, 6 and 7. Figure 5 present the evolution of the output $y(k)$ and the desired reference trajectory $r(k)$, figure 6 illustrate the evolution of the controller $u(k)$ and figure 7 shows the evolution of the sliding surface $S(k)$.

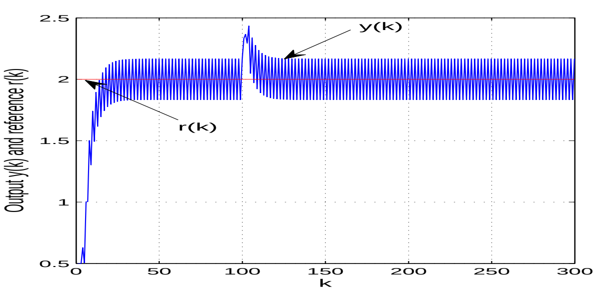

Fig. 5. Evolution of the output $y(k)$ and the desired reference trajectory $r(k)$ DSMC without external disturbances

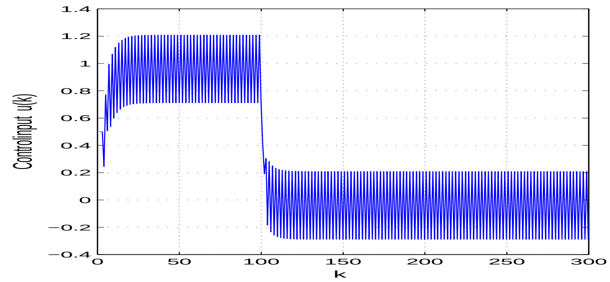

Fig. 6. Evolution of control input $u(k)$ DSMC without external disturbances

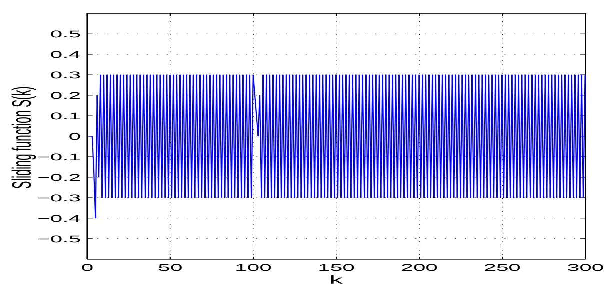

Fig. 7. Evolution of the sliding function $S(k)$ DSMC without external disturbances

It can be seen that the classical sliding mode control is able to remove the external disturbances but it can not eliminate the chattering phenomenon.

IV. Disrete Second Order Sliding Mode Control

The main drawback of the discrete sliding mode control is the chattering phenomenon caused by the discontinuous term. In order to overcome this problem, we propose a new the discrete second order sliding mode control for single-input single-output systems with input-output model (2-DSMC). Consider the system defined by the equation (1).
To obtain a discrete second order sliding mode control, the sliding function must verify the two following conditions:

$$\left\{ \begin{array}{l} S\left(k+1\right) = 0 \\ S\left(k\right) = 0 \end{array} \right. \tag{9}$$

So, we consider a new system whose variables are $S\left(k+1\right)$ and $S\left(k\right)$.
In the case of second order sliding mode control, the sliding function is selected as follows [5], [13]:

$$\sigma\left(k\right) = S\left(k\right) + \beta\, S\left(k-1\right) \tag{10}$$

where

$$\left\{ \begin{array}{c} S\left(k\right) = C\left(q^{-1}\right)\left(y\left(k\right) - r\left(k\right)\right) = C\left(q^{-1}\right)e\left(k\right) \\ 0 < \beta < 1 \end{array} \right.$$

The equivalent control law that ensures ideal sliding mode is deduced from the following equation:

$$\sigma\left(k+1\right) = \sigma\left(k\right) = 0 \tag{11}$$

We have

$$S(k+1) = B\left(q^{-1}\right) E\left(q^{-1}\right) u(k) + G\left(q^{-1}\right) y(k) \\ -C\left(q^{-1}\right) r(k+1)$$

Then, $\sigma(k+1)$ can be written as:

$$\begin{aligned} \sigma(k+1) &= S(k+1) + \beta S(k) \\ &= B\left(q^{-1}\right) E\left(q^{-1}\right) u(k) + G\left(q^{-1}\right) y(k) \\ &+E\left(q^{-1}\right) d(k+1) - C\left(q^{-1}\right) r(k+1) + \beta S(k) \end{aligned}$$

Using this last relation and the equation (11), we can obtain the equivalent control law as:

$$u_{eq_2}(k) = \left[B\left(q^{-1}\right) E\left(q^{-1}\right)\right]^{-1} \begin{bmatrix} -\beta S(k) - G\left(q^{-1}\right) y(k) \\ +C\left(q^{-1}\right) r(k+1) \end{bmatrix} \tag{12}$$

with $G(q^{-1})$ is the solution of the diophantine polynomial equation (4).
In the case of discrete second order mode control, the discontinous term $u_{dis_2}(k)$ is defined by [13]:

$$u_{dis_2}(k) = u_{dis_2}(k-1) - T_e M' \, sign(\sigma(k)) \tag{13}$$

with T_e is the sampling rate.
Then, the global control law is written as:

$$u(k) = u_{eq_2}(k) + u_{dis_2}(k) \tag{14}$$

V. Simulation Example

To evaluate the robustness of the proposed control law in the presence of disturbances, we consider the single-input single-output system used previously (section III).
The synthesis parameters are chosen as:

$$M' = 0.1, \qquad \beta = 0.1, \qquad T_e = 0.01$$

The reference input is chosen as:

$$r(k) = 2$$

A. Case 1: without external disturbances $d(k) = 0$

The simulation results of second order sliding mode control scheme are shown in figures 8, 9 and 10. Figure 8 given the evolution of the output $y(k)$ and the desired reference trajectory $r(k)$. Figure 9 shows the evolution of the controller $u(k)$. The evolution of the sliding surface $\sigma(k)$ is presented in figure 10.

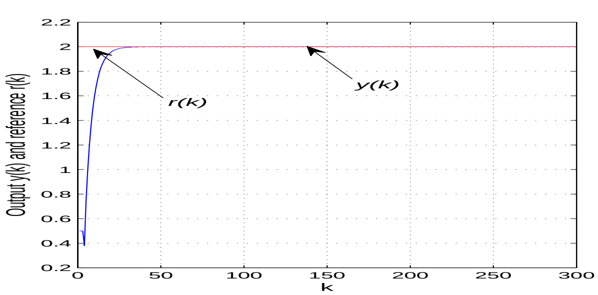

Fig. 8. Evolution of the output $y(k)$ and the desired reference trajectory $r(k)$ 2-DSMC without external disturbances

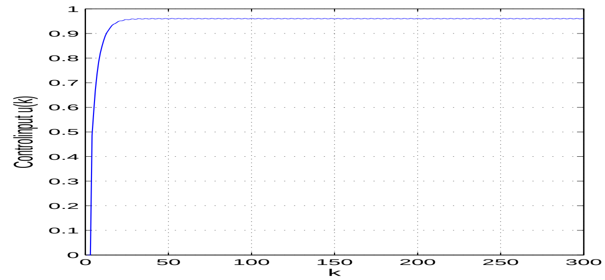

Fig. 9. Evolution of control input $u(k)$ 2-DSMC without external disturbances

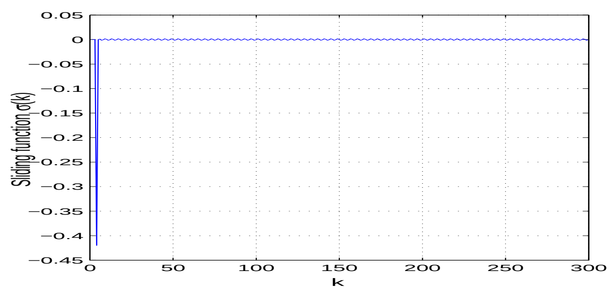

Fig. 10. Evolution of the sliding function $\sigma(k)$ 2-DSMC without external disturbances

B. Case 2: with external disturbances

In this case, the system is subjected to the same disturbances given in section III.

$$d(k) = 0.5 \qquad\qquad if \quad k > 100$$

The simulation results of second order sliding mode control are shown in figures 11, 12 and 13. Figure 11 shows the evolution of the output $y(k)$ and the desired reference trajectory $r(k)$. Figure 12 illustrate the evolution of the controller $u(k)$. The evolution of the sliding surface $\sigma(k)$ is presented in figure 13.

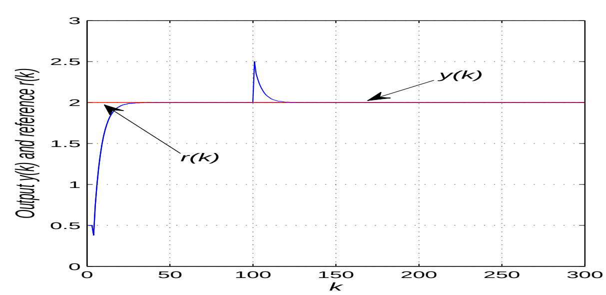

Fig. 11. Evolution of the output $y(k)$ and the desired reference trajectory $r(k)$ 2-DSMC with external disturbances

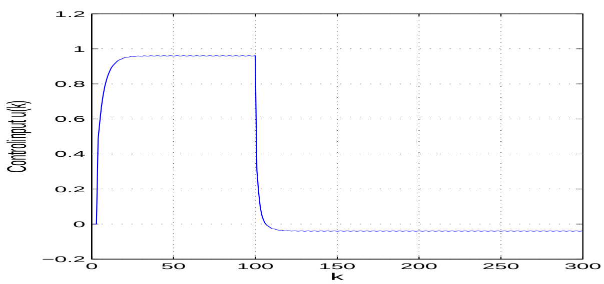

Fig. 12. Evolution of control input $u(k)$ 2-DSMC with external disturbances

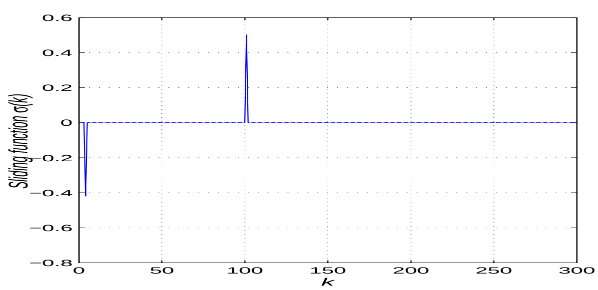

Fig. 13. Evolution of the sliding function $\sigma(k)$ 2-DSMC with external disturbances

From these figures, it is clear that the oscillations encountered in the case of the classical sliding mode control are eliminated. Therefore, the proposed discrete second order sliding mode control law is able to eliminate the chattering phenomenon.

The simulation results of the second order sliding mode control are compared with the classical sliding mode control. The result are shown in figure 14. It can be seen that the discrete second order sliding mode control is able to eliminate the chattering phenomenon that appears in the classical sliding mode control.

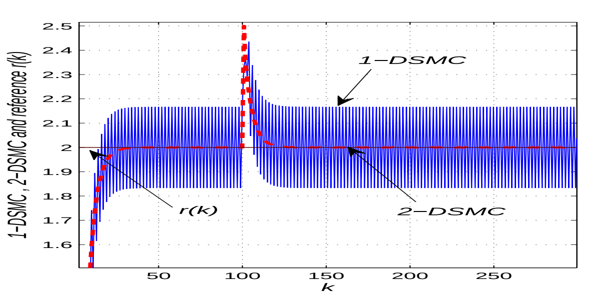

Fig. 14. Evolution of the output $y(k)$ and the desired reference trajectory $r(k)$

VI. Conclusion

In this paper, we have proposed a new discrete second order sliding mode control for the single-input single-output systems in the presence of external disturbances. This control is based on an input-output model. The key advantage of the new discrete second order sliding mode control is its ability to reject the disturbances and to reduce chattering phenomenon. A comparison between the proposed discrete second order sliding mode control and the classical sliding mode control shows the effectiveness of the proposed control strategy.

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